Proceedings of the 1999 Particle Accelerator Conference, New York, 1999

LONGITUDINAL INSTABILITY IN A 50GEV×50GEV MUON COLLIDER RING *

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Abstract

Simulations of the longitudinal dynamics in a 50GeV×50GeV muon collider ring have been performed. Operation of the ring close to the slippage factor $\eta_1 \simeq 10^{-6}$, such that synchrotron motion is frozen, minimizes the need for rf to maintain the bunch length. Without appropriate rf compensation, however, the bunch wake induces an intolerable head-to-tail energy spread. This paper demonstrates that the bunch wake may be compensated by two rf cavities with low rf voltages. With this rf setup the small energy spread of the beam ($\delta E/E$=3$\times10^{-5}$) in the 50GeV×50GeV muon collider ring can be maintained during the 1000 turn lifetime of the muons. These studies were made at the nominal design point, and sensitivities to errors were explored. The simulation also demonstrates that the computation of the wake field using bins of variable width (each with a constant number of macroparticles) accurately reproduces the wake and yields reduced computing time compared to the evaluation of the wake as the direct sum over the wakes of all preceding macroparticles.

1 INTRODUCTION

This paper presents investigations of the longitudinal dynamics in a 50GeV×50GeV muon collider ring. The critical design parameters of a 50GeV×50GeV muon collider ring, from the viewpoint of collective effects, are: The bunch has a large charge (N=4$\times10^{12}$), the bunch length is long (σ_z=13cm) compared to the pipe radius(b), the beam energy spread is very small (σ_δ=3$\times10^{-5}$), as is the slippage factor, ($\eta_1 \simeq -10^{-6}$). The muon has a life time, $\tau_\mu \simeq 1.1ms$ at 50 GeV, corresponding to 1000 turns in the ring with a circumference(C) of 300 meters. The need to minimize rf voltage leads to η_1= -10^{-6} and a synchrotron oscillation period much longer than the storage time. The small slippage over the storage time leads to dynamics similar to that in a linac. The large bunch charge induces, through the wakefield, an undesirable head-to-tail energy spread. Maintaining an intense beam with a low energy spread provides a challenge to the ring design.

We show a means of controlling the longitudinal dynamics in the 50GeV muon collider ring. That is, to limit rf, one operates the ring close to the transition (η_1=-10^{-6}) such that the synchrotron motion is frozen in the storage time, and uses two rf cavities to compensate for the ring impedance arising from beam and ring structures. Since the beam intensity decreases, due to the muon decay, rf voltages must also vary in time.

The utility of the simulation depends on the ability to calculate a sufficiently accurate wake without excessive computation. In the present code, the wake is calculated by summing the wakes from bins of variable bin width in front of the particle and δ-wakes from preceding particles in the same bin. We tested this method of calculating wakes, and found that it gives the desired accuracy and a substantial reduction in computing time when compared to wake calculations using unbinned macroparticles [1]. A more complete description of this work may be found in Ref.2 [2].

2 THE COMPUTATION OF THE WAKEFIELD AND MACROPARTICLE EQUATIONS

The wake generated by a beam interacting with discontinuities of components in the ring is approximated by a broad-band impedance. The longitudinal wake function $W_0'(z)$ for a broad-band impedance is given by [3]

$$W_0'(z) = \begin{cases} 0 & \text{if z} > 0 \\ \alpha R_s & \text{if z = 0} \\ 2\alpha R_s e^{\alpha z/c}(cos\frac{\bar\omega z}{c} + \frac{\alpha}{\bar\omega} sin\frac{\bar\omega z}{c}) & \text{if z} < 0, \end{cases} \tag{1}$$

where $\alpha = \omega_R/2Q$ and $\bar\omega = \sqrt{w_R^2 - \alpha^2}$, Q=$R_s\sqrt{C/L}$ is the quality factor, $w_R = 1/\sqrt{LC}$ is the resonant frequency and R_s is the shunt impedance. For the broadband model, $Q = 1$ and $\omega_R = c/b$. The more common impedance $Z_{||}/n_h$=$(2\pi b/C)R_s$. Here, n_h is the harmonic number.

The variable bin size technique mentioned above works as follows. Firstly, the bunch is sliced into longitudinal bins with a constant number of particles, N_B, in each bin. This results in bins of variable bin width. The wake is then expressed, for the i^{th} particle, assumed to be in bin I, as

$$W(z_i(n)) = -\frac{N_p r_o}{N_m \gamma}[\sum_J^{I<J} N_B W_o'(z_I(n) - z_J(n)) - \sum_j^{i<j} W_o'(z_i(n) - z_j(n)) - W_o'(0)], \tag{2}$$

where N_p is number of particles in the bunch and N_m is the number of macroparticles. The lower case indices refer to particles in the same bin, while the upper case indices refer to different bins. The wake from macroparticles in preceding bins ($J > I$) is calculated (the first term on the RHS

* Work supported by the U.S. Department of Energy under contract No. EDDEFG-03-95ER-40936 and DE-AC03-76SF00098

of Eq.(2)) as arising from a *single* macroparticle located at z_J, the center of bin J. The interaction between macroparticles in the same bin is included as δ-function wakes (the second term on the RHS of Eq.(2)), and finally $-W_o'(0)$ is the wake generated by the macroparticle itself.

We used of order 2000 bins for 44000 macroparticles in the simulation. The wake field that was in good agreement with analytical results. The numbers of bins and macroparticles must be properly chosen, as they depend on ring parameters such as the bunch length and the radius of the beam pipe.

Macroparticles, with an initially Gaussian distribution, are tracked in phase space with equations of motion which include kicks by two rf cavities, a longitudinal wake kick and a drift that depends on the momentum compaction. Each macroparticle i has position and energy coordinates (z_i, δ_i) and is tracked for 1000 turns. The longitudinal difference equations for the i'th macroparticle at revolution number n is derived from its coordinates on turn $n-1$ by:

$$\delta_i(n) = \delta_i(n-1) + K_{rf}(n-1) + W(z_i(n)), \qquad (3)$$

$$z_i(n) = z_i(n-1) + (\eta_1\delta + \eta_2\delta^2 + \eta_3\delta^3)C. \qquad (4)$$

Here, z is the longitudinal coordinate with respect to the bunch center, $\delta = \delta P/P$ is the relative momentum error of the particle, η_1, η_2, η_3 are the linear and higher order momentum compaction parameters. The rf impulse $K_{rf}(n-1)$ due to the two rf voltages is given by

$$K_{rf}(n-1) = \frac{eV_{rf1}e^{-\frac{T_o(n-1)}{\gamma\tau_\mu}}}{E_o} sin(\frac{w_{rf1}}{c}z_i + \phi_1) + \frac{eV_{rf2}e^{-\frac{T_o(n-1)}{\gamma\tau_\mu}}}{E_o} sin(\frac{w_{rf2}}{c}z_i + \phi_2). \qquad (5)$$

An example of rf parameters that compensate the wake are shown in Table 1. The factor $e^{-\frac{T_0(n-1)}{\gamma\tau_\mu}}$ in the rf voltages is introduced to compensate for the decreasing beam intensity due to muon decay. Here, T_0 is the revolution period.

Table 1: Rf parameters used in the simulation.

	One cavity	**Two cavities**
rf frequency f_{rf} (MHz)	570	823 and 399
harmonic number(n_h)	570	823 and 399
rf voltage V_{rf} (KV)	14.1	4.26 and 12.12
phase offset ϕ(radian)	3.55	3.755 and 3.415

2.1 Numerical example of the compensation of the bunch wake

Figs.1(a) and (b) show the beam phase space after 1 turn and 1000 turns in the case that one cavity in Table 1 has been choosen to minimize the induced energy spread, respectively. One cavity compensates for the wake in the center of the bunch. After 1000 turns, the tail particles gain or lose energy from the large rf kick at the longitudinal positions where the bunch wake becomes small, as seen in Fig.1(b).

Fig.2(a) show the beam phase space after 1000 turns when two cavities in Table 2 are applied. The beam distribution after 1000 turns is not distorted by the sum of the rf voltages and the bunch wake and the beam distribution remains intact. This can be understood from examination of Fig.2(b), where the rf voltage (curve (1)), bunch wake voltage (curve (2)), and the resulting total voltage (curve (3)) are plotted after 1000 turns. Fig.2(b) shows the cancellation of the wake by the rf voltages. We note that the bunch wake in the 50GeV×50GeV muon collider ring can be compensated by very low rf voltages.

3 SENSITIVITY STUDIES OF LONGITUDINAL DYNAMICS

3.1 Sensitivity to beam current and $Z_{||}/n_h$

We investigated the beam phase space with beam current varying from its design value by -10%,-5%,+5% and +10%. The rf parameters are fixed. The energy spread has increased roughly 8% for variation of ± 10%, and 2% for ± 5 % after 1000 turns.

We also investigated the beam phase space for variations of the impedance: $Z_{||}/n_h$ = 0.1Ω, 0.5Ω, 0.7Ω and 1Ω. Voltages in the rf cavities are varied proportionally to the magnitude of $Z_{||}/n_h$. The potential well distortion due to the bunch wake increases with $Z_{||}/n_h$ and becomes quite noticeable at 1Ω.

3.2 Dependence of longitudinal dynamics on σ_δ and η_1, η_2, η_3

Fig.3(a) shows the beam phase space with σ_δ=10^{-3} and η_1= -10^{-2}. When $\delta'(s)$ integrated over 10^3 turns is small compared to σ_δ, the motion of a particle is determined by the magnitude of η_1. The large slippage dominates the motion and leads to streaming in z motion, as is seen in Fig.3(a). Fig.3(b) shows the beam phase space in the case of σ_δ= 3×10^{-5} and η_1= -10^{-2}. In the case of large η_1, and small energy spread, the dynamics is significantly more complicated, showing energy spread and streaming in z, as shown in Fig.3(b). In the calculations for Figs.3, we have had η_2=η_3=0 and the equation of motion is $z' = \eta_1\delta(s)$, where $z' = \frac{dz}{ds}$ and s is distance traveled in the ring. The resultant phase space after 10^3 turns is summarized as a function of (η_1, σ_δ) in Fig.4. Regions (a) and (b) in Fig.4 were covered in Figs.(1).

On the other hand, the two rf voltages compensation reduces the phase space distortions of the beam for the (a), (b) and (d) regions of Fig.4. The motion in region (c) of Fig.4 is dominated by slippage, and the rf does not affect the dynamics. In summary, two rf cavities can be used to remove the beam tail of region (b) of Fig.4 and the energy spread of region (d) of Fig.4.

Longitudinal dynamics for varying η_2 and fixed η_1=-10^{-6} and η_2=0 has been examined. The z' equation includes a nonlinear term, z'=$(\eta_1 + \eta_2\delta(s))\delta(s)$. With two rf cavities used to compensate the bunch wake, the energy does not show significant change after 1000 turns, and the motion is a nonlinear streaming with the energies taken as their initial values. We see this in Fig.5(a) with η_2=100, where the energy spread of the macroparticles is basically unchanged. The nonlinearity of the streaming is noticeable for particles with $\eta_2 > \eta_1/\delta(s)$. The behavior of the phase space parameterized by η_2 and σ_δ is summarized in Fig.5(b).

Longitudinal dynamics for varying η_3 and fixed η_1=-10^{-6} and η_2=0 have also been examined. The energy evolution is small due to good compensation, and the position evolves according to $z' = (\eta_1 + \eta_3\delta^2(s))\delta(s)$, where the energy can be taken roughly to be fixed at its initial value. The nonlinear coefficient η_3 becomes important when $\eta_3 > \eta_1/\delta^2(s)$. One case with η_3=100 is shown in Fig.6(a). The parameter space for varying η_3 and σ_δ is summarized in Fig.6(b).

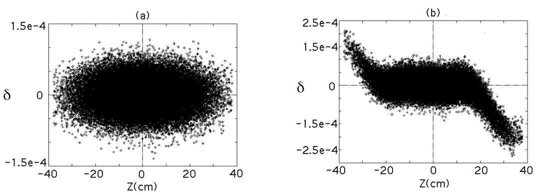

Figure 1: Longitudinal beam phase space at (a) turn 1 and (b) turn 1000 when one rf cavity in Table 1 is applied. The tails of the bunch are distorted by the partly compensated rf voltage. σ_δ=3$\times10^{-5}$, η_1= -1$\times10^{-6}$ and $\left|\frac{Z_{||}}{n_h}\right|$=0.1$\Omega$.

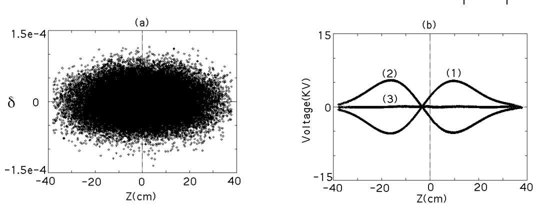

Figure 2: (a) longitudinal beam phase space after 1000 turns when two rf voltages in Table 1 are applied. (b) shows voltages due to the two rf cavities (curve(1)), the bunch wake (curve(2)), and the net voltage (curve(3)) after 1000 turns. The bunch wake is well-compensated by the two rf cavities. σ_δ=3$\times10^{-5}$, η_1= -1$\times10^{-6}$ and $|Z_{||}/n_h|$=0.1Ω.

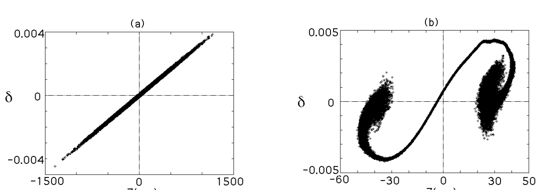

Figure 3: The longitudinal phase space after 1000 turns for different values of σ_δ and η_1. One rf voltage in Table 1 is used to compensate for the bunch wake. σ_δ=10^{-3} and η_1= -0.01 in (a); σ_δ=3$\times10^{-5}$ and η_1= -0.01 in (b).

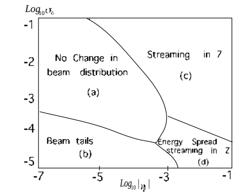

Figure 4: A summary of the beam dynamics in σ_δ and η_1 parameter space. One rf voltage in Table 1 is used to compensate for bunch wake. $|Z_{||}/n_h|$=0.1Ω.

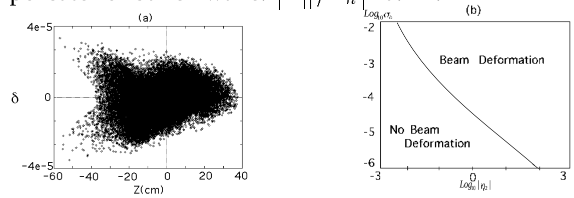

Figure 5: (a) the longitudinal phase space after 1000 turns for η_2=100. Two cavities are used to compensate for bunch wake. η_1= -10^{-6}, η_3=0, σ_δ=3$\times10^{-5}$ and $|Z_{||}n_h/|$=0.1Ω. (b) shows beam dynamics in the σ_δ and η_2 parameter space. Right upper parameter space shows the beam streaming.

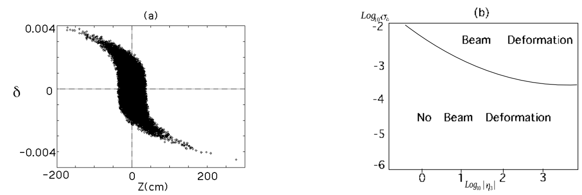

Figure 6: (a) the longitudinal phase space after 1000 turns with η_3=100. η_1=-10^{-6}, η_2=0, σ_δ=10^{-3} and $\left|\frac{Z_{||}}{n_h}\right|$=0.1$\Omega$. (b) shows beam dynamics in the σ_δ and η_3 parameter space. Right upper parameter space shows the beam streaming.

4 CONCLUSION

Longitudinal motion in the 50GeV$\times$50GeV muon collider ring is investigated with a multi-particle tracking code. A binning scheme is used to enhance the computational efficiency of the simulation and muon decay is included. The operation of a ring with small η_1, so that the synchrotron oscillation is frozen during the storage time, and with the bunch wake compensated by two low rf cavities, is studied.

The longitudinal dynamics is seen to be controllable with proper choice of rf parameters. One cavity can be used to control the motion of the core of the bunch, while a second controls the tails. We studied the role played by important ring parameters in the longitudinal dynamics. Longitudinal motion with compensation of the wake was studied with various slippage factors (η_1, η_2 and η_3). The sensitivity of the compensation scheme to variations in ring parameters was examined.

5 REFERENCES

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